PRACTICE LESSON 3.2

EXERCISE 1

(X) - extraversion
(Y) emotional stability

PART.	(X)	(Y)	XY	X^2	Y^2	$\hat{Y}$ (d)	e
1	5	6	30	25	36	6.532	-0.532
2	10	6	60	100	36	5.737	0.263 — best adjusted
3	4	3	12	16	9	6.691	-3.691 — worst adjusted
4	7	8	56	49	64	6.214	1.786
5	6	6	36	36	36	6.373	-0.373
6	5	3	15	25	9	6.532	-3.532
7	4	5	20	16	25	6.691	-1.691
8	4	9	36	16	81	6.691	2.309
9	4	10	40	16	100	6.691	3.309
10	3	9	27	9	81	6.85	2.15
	52	65	332	308	477		

(a) $\hat{Y} = a + bX = 7.327 - 0.159X$

$$b = r_{XY} \cdot \frac{S_Y}{S_X} = -0.132 \cdot \frac{2.334}{1.939} = -0.159$$

$$r_{XY} = \frac{\frac{\Sigma XY}{N} - \bar{X} \cdot \bar{Y}}{S_X S_Y} = \frac{\frac{332}{10} - 5.2 \cdot 6.5}{1.939 \cdot 2.334} = \frac{33.2 - 33.8}{4.526} = \frac{-0.6}{4.526} = -0.132$$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{52}{10} = 5.2$$

$$\bar{Y} = \frac{\Sigma Y}{N} = \frac{65}{10} = 6.5$$

$$S_X = \sqrt{\frac{\Sigma X^2}{N} - \bar{X}^2} = \sqrt{\frac{308}{10} - 5.2^2} = \sqrt{3.76} = 1.939$$

$$S_Y = \sqrt{\frac{\Sigma Y^2}{N} - \bar{Y}^2} = \sqrt{\frac{477}{10} - 6.5^2} = \sqrt{5.45} = 2.334$$

$$a = \bar{Y} - b\bar{X} = 6.5 - (-0.159) \cdot 5.2 = 7.327$$

(b) $\hat{Y} = 7.327 - 0.159 \cdot 0 = 7.327 \rightarrow a$

(c) $-0.159 = b$

EXERCISE 2

$\bar{X} = 6$

$\bar{Y} = 7$

$\Sigma (X-\bar{X})^2 = 360$

$\Sigma (Y-\bar{Y})^2 = 240$

$\Sigma (X-\bar{X})(Y-\bar{Y}) = 250$

RAW SCORES:

$$b = \frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{\Sigma (X-\bar{X})^2} = \frac{250}{360} = 0.694$$

$$a = \bar{Y} - b\bar{X} = 7 - 0.694 \cdot 6 = 7 - 4.167 = 2.833$$

$$\hat{Y} = a + bX$$

$$\boxed{\hat{Y} = 2.833 + 0.694X}$$

DIFFERENTIAL SCORES:

$$\hat{y} = bx \qquad \boxed{\hat{y} = 0.694x}$$

STANDARD SCORES:

$$z_{\hat{Y}} = r_{XY} z_X \qquad \boxed{z_{\hat{Y}} = 0.852 z_X}$$

$$r_{XY} = \frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{\sqrt{\Sigma (X-\bar{X})^2}\sqrt{\Sigma (Y-\bar{Y})^2}} = \frac{250}{\sqrt{360}\cdot\sqrt{240}} = \frac{250}{18.974 \cdot 15.492} = \frac{250}{293.94} = 0.85$$

EXERCISE 3

a TRUE
b FALSE
c FALSE
d TRUE
e TRUE

EXERCISE 4

RAW SCORES:

$\hat{Y} = a + bX$

STUDENT 1 → $4.367 = a + b \cdot 5$

STUDENT 2 → $6.387 = a + b \cdot 8$

$$\begin{cases} -4.367 = -a - 5b \\ +6.387 = +a + 8b \end{cases}$$

$$2.02 = 3b$$

$$\frac{2.02}{3} = b \rightarrow b = 0.673$$

$6.387 = a + 0.673 \cdot 8$

$6.387 = a + 5.384$

$1.003 = a$

$$\boxed{\hat{Y} = 1.003 + 0.673X}$$

DIFFERENTIAL SCORES: $\hat{y} = bx \rightarrow \boxed{\hat{y} = 0.673x}$

STANDARD SCORES: $z_{\hat{Y}} = r_{XY} z_X \rightarrow \boxed{z_{\hat{Y}} = 0.862 z_X}$

STUDENT 1: $0.025 = r_{XY} \cdot 0.029$

$$\frac{0.025}{0.029} = r_{XY} \rightarrow r_{XY} = 0.862$$

EXERCISE 5

(a) $\hat{y} = bx \rightarrow \boxed{\hat{y} = 8.356x}$

$b = r_{xy} \cdot \frac{S_y}{S_x} = 0.798 \cdot \frac{17.476}{1.669} = 0.798 \cdot 10.471 = 8.356$

$R^2_{xy} = \frac{SS_{exp}}{SS_T} = \frac{1362.513}{2138} = 0.637 \rightarrow r_{xy} = \sqrt{0.637} = 0.798$

$x = X - \bar{X} = 3 - 2.75 = 0.25$

$\hat{y} = 8.356 \cdot 0.25 = 2.089$

(b) $b = 8.356$

(c) Yes F $F_t = F(\alpha, k, N-k-1) = F(0.05, 1, 6)$

$10.542 > 5.99$ — ~~1/6~~

EXERCISE 6

$N = 50$
$\bar{y} = 5.3$
$\bar{X} = 2.5$
$a = 2$
$b = ?$

$\bar{y} = a + b\bar{X}$
$5.3 = 2 + b \cdot 2.5$
$3.3 = 2.5b$
$\frac{3.3}{2.5} = b$
$\boxed{1.32 = b}$

EXERCISE 7

① k = number of independant variables

	SS	df	$MS = S^2$	F	sig
REG	198	1	198	18	.003
RES	88	8	11		
TOTAL	286				

N-k-1

② $\frac{198}{S^2_{res}} = 18 \rightarrow 198 = 18\,S^2_{res} \rightarrow \frac{198}{18} = S^2_{res}$

$11 = S^2_{res}$

③ $S^2_{res} = \frac{SC_{res}}{N-k-1}$

$11 = \frac{SC_{res}}{8} \rightarrow SC_{res} = 11 \cdot 8 = 88$

EXERCISE 8

X	X^2
1	1
2	4
3	9
4	16
5	25
15	55

(a) $SS_{exp} = b^2 \cdot N \cdot S_x^2 = 1.2^2 \cdot 5 \cdot 2 = \boxed{14.4}$

$$S_x^2 = \frac{\Sigma X^2}{N} - \bar{X}^2 = \frac{55}{5} - 3^2 = 11 - 9 = 2$$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{15}{5} = 3$$

(b) $$R^2 = \frac{SS_{exp}}{SS_T} \rightarrow 0.643^2 = \frac{14.4}{SS_T}$$

$$0.413\ SS_T = 14.4$$

$$SS_T = \frac{14.4}{0.413} = \boxed{34.867}$$