

Psychometrics. Partial 2. January 2020. Type A / Type C

Case 1.

1.

Test A

Pass
Fail

		Test B		
		Pass	Fail	
8 (a)	2 (b)	10 (g)		
0 (c)	2 (d)	2 (h)		
8 (e)	4 (f)	12 (N)		

1	2	3	4	5	6	7	8	9	10	11	12
b	a	a	a	a	a	d	a	a	d	a	b

Hamberton & Novick

$$P_0 = \frac{fc}{N} = \frac{a+d}{N} = \frac{10}{12} = 0.83$$

$$F_a = \frac{e \cdot g}{N} + \frac{f \cdot h}{N} = \frac{80}{12} + \frac{8}{12} = 7.33$$

$$2. K = \frac{F_c - F_a}{N - F_a} = \frac{10 - 7.33}{12 - 7.33} = \frac{2.67}{4.67} \approx 0.57 < 0.7 \text{ Inappropriate}$$

Case 2

$$3. T = 50 + 10 \sum x \Rightarrow T = 50 + 10(-1) = 40$$

$\bar{x} = 10$
$s_x = 3$
$x = 7$

$$z = \frac{x - \bar{x}}{s_x} \quad z = \frac{7 - 10}{3} = -1$$

$$4. y = a + b \sum x$$

$$y = 20 + 3(-1) = 17$$

$$5. \sum x = -1 \rightarrow p = 0.1587 \text{ (obtained from Z scores table)}$$

$$p \approx 16$$

$$6. r_{xy} = 0.7 \quad \sum \hat{y} = r_{xy} \cdot \sum x \Rightarrow \sum \hat{y} = 0.7 \cdot 1 = 0.7$$

$$\sum x = 1$$

$$E = 5 + 2 \sum z_n$$

$$E = 5 + 2(0.7) = 5 + 1.4 = 6.4 \approx 6$$

$$7. x = 6 \quad x^* = y = \left(\frac{s_y}{s_x} \right) (x - \bar{x}) + \bar{y}$$

$$\bar{x} = 6$$

$$s_x^2 = 9 \quad s_x = 3$$

$$\bar{y} = 10$$

$$s_y^2 = 4 \quad s_y = 2$$

$$y = \left(\frac{2}{3} \right) (0) + 10 = 10$$

Case 3.

$$N = 100$$

$$EI = 20$$

$$\bar{x} = 6$$

$$s_x^2 = 9 \rightarrow s_x = 3$$

$$\frac{s_T^2}{s_x^2} = 0.7 = r_{xx'}$$

$$\sum x = 1.5$$

$$C.L. = 99\% \rightarrow Z_c = 2.58$$

$$C.L. = 95\% \rightarrow Z_c = 1.96$$

$$8. \sum x = \frac{x - \bar{x}}{s_x} \Rightarrow 1.5 = \frac{x - 6}{3} \Rightarrow x = 4.5 + 6 = 10.5$$

$$Lim = T' \pm E_{max} = 9.15 \pm 3.59 < \begin{matrix} 12.74 \\ 5.56 \end{matrix}$$

$$T' = r_{xx'}(x - \bar{x}) + \bar{x} = 9.15$$

$$T' = 0.7(10.5 - 6) + 6 = [0.7(4.5)] + 6 = 3.15 + 6 = 9.15$$

$$E_{max} = Z_c \cdot S_{Tx} = 2.58 \cdot 1.39 = 3.59$$

$$S_{Tx} = s_e \cdot \sqrt{r_{xx'}} = 1.65 \sqrt{0.7} = 1.65 \cdot 0.84 = 1.39$$

$$s_e = s_x \cdot \sqrt{1 - r_{xx'}} = 3 \cdot \sqrt{1 - 0.7} = 3 \cdot \sqrt{0.3} = 3 \cdot 0.55 = 1.65$$

$$9. Lim = T' \pm E_{max} = 3.15 \pm 3.59 < \begin{matrix} 6.74 \\ -0.44 \end{matrix}$$

Otro modo

$$T' = T - \bar{T} = 9.15 - 6 = 3.15$$

$$\bar{T} = \bar{x}$$

$$T' = r_{xx'} \cdot x_i = 0.7 \cdot 4.5 = 3.15$$

$$x_i = x - \bar{x} = 10.5 - 6 = 4.5$$

igual que en puntuaciones directas

$$10. Lim = \sum T' \pm E_{max} = 1.26 \pm 0.90 < \begin{matrix} 2.16 \\ 0.36 \end{matrix}$$

$$\sum T' = r_{Tx} \cdot \sum x = 0.84 \cdot 1.5 = 1.26$$

$$r_{Tx} = \sqrt{r_{xx}} = \sqrt{0.7} = 0.84$$

$$\sum x = 1.5$$

$$E_{max} = Z_c \cdot S_{ZTZ_x} = 1.96 \cdot 0.46 = 0.90$$

$$S_{ZTZ_x} = \sqrt{1 - r_{xx'}} \cdot \sqrt{r_{xx'}} = \sqrt{1 - 0.7} \cdot \sqrt{0.7} = 0.55 \cdot 0.84 = 0.46$$

$$11. n = \frac{r_{xx'}(1 - r_{xx'})}{r_{xx'}(1 - 0.9)} = \frac{0.9(1 - 0.7)}{0.7(1 - 0.9)} = \frac{0.9 \cdot 0.3}{0.7 \cdot 0.1} = \frac{0.27}{0.07} \approx 3.86$$

$$n = \frac{EF}{EI} \Rightarrow 3.86 = \frac{EF}{20} \Rightarrow EF \approx 77$$

12.

$$s_1^2 = 9$$

$$s_2^2 = 4$$

$$r_{11} = 0.7$$

$$r_{22} = 1 - \frac{s_1^2}{s_2^2} (1 - r_{11})$$

$$r_{22} = 1 - \frac{9}{4} (1 - 0.7) = 1 - (2.25 \cdot 0.3) = 1 - 0.675 = 0.32$$

Case 4

$$\begin{array}{l} \bar{x} = 8 \\ s_x = 1 \end{array} \parallel \begin{array}{l} \bar{y} = 4 \\ s_y^2 = 4 \\ s_y = 2 \end{array} \parallel r_{xy}^2 = 0.64$$

13. $CPV = 1 - C.A = 1 - 0.6 = 0.4$

$$C.A = \sqrt{1 - r_{xy}^2}$$

$$C.A = \sqrt{1 - 0.64} = \sqrt{0.36} = 0.6$$

14.

$$Z_x = \frac{x - \bar{x}}{s_x}$$

$$Z_{y'} = r_{xy} Z_x = 0.8 \cdot 1 = 0.8$$

$$x = 9 \quad Z_x = \frac{9 - 8}{1} = 1$$

$$r_{xy} = \sqrt{0.64} = 0.8$$

15.

$$Lim = y' \pm \mathcal{E}_{max} = 4 \pm 3.096 \begin{array}{l} < 7.096 \\ < 0.904 \end{array}$$

$$x = 8$$

$$\alpha = 0.01$$

$$C.L = 99\%$$

$$Z_c = 2.58$$

$$y' = a + bx \Rightarrow y' = -8.8 + (1.6 \cdot 8) = 4$$

$$b = r_{xy} \cdot \frac{s_y}{s_x} = 0.8 \cdot \frac{2}{1} = 1.6$$

$$a = \bar{y} - b\bar{x} = 4 - (1.6 \cdot 8) = -8.8$$

Another way to solve:

$$y' = r_{xy} \cdot \frac{s_y}{s_x} (x - \bar{x}) + \bar{y}$$

$$y' = 0.8 \cdot \frac{2}{1} (8 - 8) + 4$$

$$y' = 4$$

$$\mathcal{E}_{max} = Z_c \cdot S_{y \cdot x} = 2.58 \cdot 1.2 = 3.096$$

$$S_{y \cdot x} = s_y \sqrt{1 - r_{xy}^2} = 2 \sqrt{1 - 0.64} = 2 \sqrt{0.36} = 2 \cdot 0.6 = 1.2$$