

# EJERCICIO 1

PART	A	B	C	D	E	F	<sup>even</sup> P	<sup>odd</sup> i	d	d <sup>2</sup>	X	X <sup>2</sup>	P <sup>2</sup>	i <sup>2</sup>
1	0	1	0	1	1	1	3	1	2	4	4	16	9	1
2	1	0	0	1	1	0	1	2	-1	1	3	9	1	4
3	1	0	0	0	0	1	1	1	0	0	2	4	1	1
4	0	1	1	1	1	0	2	2	0	0	4	16	4	4
5	0	0	0	1	0	1	2	0	2	4	2	4	4	0
6	1	1	1	1	1	1	3	3	0	0	6	36	9	9
7	0	1	1	1	1	1	3	2	1	1	5	25	9	4
8	0	0	0	1	1	1	2	1	1	1	3	9	4	1
							17	12	5	11	29	119	41	24

a) RULON:

$$r_{xx'} = 1 - \frac{S_d^2}{S_x^2} = 1 - \frac{0'98}{1'73} = 0'43$$

$$S_d^2 = \frac{\sum d^2}{N} - \bar{d}^2 = \frac{11}{8} - 0'625^2 = 0'98$$

$$\bar{d} = \frac{\sum d}{N} = \frac{5}{8} = 0'625$$

$$S_x^2 = \frac{\sum X^2}{N} - \bar{X}^2 = \frac{119}{8} - 3'625^2 = 1'73$$

$$\bar{X} = \frac{\sum X}{N} = \frac{29}{8} = 3'625$$

b) GUTMAN-FLANAGAN

$$r_{xx'} = 2 \left( 1 - \frac{S_p^2 + S_i^2}{S_x^2} \right) = 2 \left( 1 - \frac{0'61 + 0'75}{1'73} \right) = 0'43$$

$$S_p^2 = \frac{\sum p^2}{N} - \bar{p}^2 = \frac{41}{8} - 2'13^2 = 0'61$$

$$\bar{p} = \frac{\sum p}{N} = \frac{17}{8} = 2'13$$

$$S_i^2 = \frac{\sum i^2}{N} - \bar{i}^2 = \frac{24}{8} - 1'5^2 = 0'75$$

$$\bar{i} = \frac{\sum i}{N} = \frac{12}{8} = 1'5$$

# EXERCICIO 2

PART.	A	B	C	D	E	F	X	X <sup>2</sup>
1	0	0	0	0	0	0	0	0
2	1	1	0	0	1	1	4	16
3	1	1	1	0	0	1	4	16
4	0	1	1	0	1	0	3	9
5	0	0	0	1	1	1	3	9
6	1	1	1	0	0	0	3	9
7	0	1	1	1	1	0	4	16
8	1	1	1	1	1	1	6	36
9	1	1	1	1	1	1	6	36
10	1	1	1	1	0	1	5	25
P	0'6	0'8	0'7	0'5	0'6	0'6		
q	0'4	0'2	0'3	0'5	0'4	0'4		Σ
S <sub>j</sub> <sup>2</sup> = P·q	0'24	0'16	0'21	0'25	0'24	0'24		1'34
Σ							38	172

$$a) \alpha = \frac{n}{n-1} \left( 1 - \frac{\sum S_j^2}{S_x^2} \right) = \frac{6}{5} \left( 1 - \frac{1'34}{2'76} \right) = 0'62$$

$$S_x^2 = \frac{\sum X^2}{N} - \bar{X}^2 = \frac{172}{10} - 3'8^2 = 17'2 - 14'44 = 2'76$$

$$\bar{X} = \frac{\sum X}{N} = \frac{38}{10} = 3'8$$

$$r_{vx} = \sqrt{r_{xx'}} = \sqrt{0'62} = 0'79$$

$$b) r_{22} = 1 - \frac{S_1^2}{S_2^2} (1 - r_{11}) = 1 - \frac{2'76}{20} (1 - 0'62) = 0'95$$



### EXERCISE 3

$$r_{xe} = 0.6$$

$$z_x = 0.5$$

$$CL = 90\%$$

$$CI = z_T' \pm E_{max} = 0.4 \pm 0.79 < \begin{matrix} 1.19 \\ -0.39 \end{matrix}$$

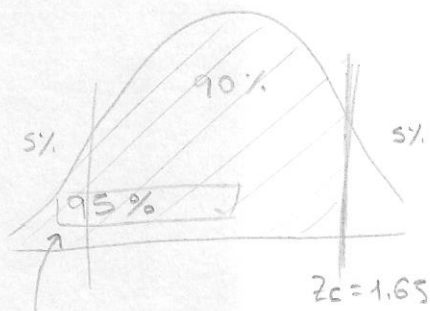
$$z_T' = r_{XT} z_x = 0.8 \cdot 0.5 = 0.4$$

Pearson's slide 9, deduction 7

$$r_{xx'} = 1 - r_{xe}^2$$

$$r_{xx'} = 1 - 0.6^2 = 1 - 0.36 = 0.64 \rightarrow r_{XT} = \sqrt{r_{xx'}} = \sqrt{0.64} = 0.8$$

$$E_{max} = z_c \cdot S_{z_T z_x} = 1.65 \cdot 0.48 = 0.79$$



You have to find in the z scores table the z value that corresponds to an area of 0.95  $\rightarrow z_c = 1.65$

$$S_{z_T z_x} = \sqrt{1 - r_{xx'}} \cdot \sqrt{r_{xx'}} = \sqrt{1 - 0.64} \cdot \sqrt{0.64} = 0.6 \cdot 0.8 = 0.48$$

EJERCICIO 4

FÓRMULA DE SPEARMAN-BROWN

$$r_{pi} = 0'44$$

$$r_{xx'} = \frac{2r_{pi}}{1+r_{pi}} = \frac{2 \cdot 0'44}{1'44} = 0'61$$

$$r_{xu} = \sqrt{r_{xx'}} = \sqrt{0'61} = 0'78$$

# EXERCICIO 5

$$\hat{\alpha}_1 = 0.62$$

$$\hat{\alpha}_2 = 0.7$$

$$r_{x_1x_2} = 0.89$$

$$nc \ 95\% \ (\alpha = 0.05)$$

$$t = \frac{(\hat{\alpha}_1 - \hat{\alpha}_2) \sqrt{N-2}}{\sqrt{[4(1-\hat{\alpha}_1)(1-\hat{\alpha}_2)(1-r_{x_1x_2}^2)]}}$$

$$t = \frac{(0.62 - 0.7) \sqrt{10-2}}{\sqrt{[4(1-0.62)(1-0.7)(1-0.89^2)]}}$$

$$t = \frac{(-0.08 \cdot 2.83)}{\sqrt{(4 \cdot 0.38 \cdot 0.3 \cdot 0.21)}} = \frac{-0.23}{\sqrt{0.1}} = \frac{-0.23}{0.32} = -0.72$$

$$t(\alpha, N-2) = t(0.05, 8) = 2.306$$

$|-0.72| < 2.306 - H_0$  La diferencia entre ambos coeficientes no es estadísticamente significativa.

## EXERCICIO 6

a) Items dicotómicos  
- Diferente dificultad } KR20

PART	A	B	C	D	X	X <sup>2</sup>
1	1	1	0	1	3	9
2	1	1	1	1	4	16
3	1	0	0	0	1	1
4	0	1	1	1	3	9
5	1	1	0	0	2	4
6	1	1	0	1	3	9
P	$\frac{5}{6} = 0.83$	$\frac{0.83}{6} = 0.14$	$\frac{2}{6} = 0.33$	$\frac{4}{6} = 0.67$		
q	0.17	0.17	0.67	0.33		
P.q	0.14	0.14	0.22	0.22		
					16	48

$$KR_{20} = \frac{n}{n-1} \left( 1 - \frac{\sum Pq}{S_x^2} \right) = \frac{4}{4-1} \left( 1 - \frac{0.72}{0.87} \right) = 0.23$$

$$S_x^2 = \frac{\sum X^2}{N} - \bar{X}^2 = \frac{48}{6} - 2.67^2 = 0.87$$

$$\bar{X} = \frac{\sum X}{N} = \frac{16}{6} = 2.67$$

$$r_{xy} = \sqrt{r_{xx}} = \sqrt{0.23} = 0.48$$

$$\varepsilon = 0.72$$



6

$$r_{x_1 x_2} = 0'6$$

$$n = 2$$

muestras relacionadas  
(mismas personas  
cumplimental el  
cuestionario original y el duplicado)

$$R_{xx'} = \frac{n \cdot r_{xx'}}{1 + (n-1) r_{xx'}} = \frac{2 \cdot 0'23}{1 + (2-1) \cdot 0'23} = \frac{0'46}{1'23} = 0'37$$

$$t = \frac{(\hat{\alpha}_1 - \hat{\alpha}_2) \sqrt{N-2}}{\sqrt{[4(1-\hat{\alpha}_1)(1-\hat{\alpha}_2)(1-r_{x_1 x_2}^2)]}} = \frac{(0'23 - 0'37) \sqrt{6-2}}{\sqrt{[4(1-0'23)(1-0'37)(1-0'6^2)]}} = \frac{-0'28}{1'1} = -0'25$$

$$t(\alpha, n-2) = t(0.05, 4) = 2'78$$

temp ttea

$$|-0'25| < 2'78$$

— Ho

No hay diferencias estadísticamente signi-  
ficativas entre los 2 coeficientes de fiabilidad

### EJERCICIO 7

$$n = 150$$

$$N = 250$$

$$\bar{X} = 25$$

$$S_x^2 = 42$$

$$r_{xx'} = ?$$

KR21 < items dicotómicos  
misma dificultad

$$KR_{21} = \frac{n}{n-1} \left( 1 - \frac{\bar{X}^2}{S_x^2} \right) = \frac{150}{150-1} \left( 1 - \frac{25^2}{42} \right) = 0'51$$

### EJERCICIO 8

$$n_n = 10 \quad \left\{ \begin{array}{l} n = 25 \\ n_v = 15 \end{array} \right.$$

$$S_x^2 = 30$$

$$S_n^2 = 10$$

$$S_v^2 = 11$$

(se espera  $\alpha < \beta$  porque  $\alpha$  infravalora el coeficiente de fiabilidad  
cuando el nº de items en cada subtest es diferente)  
varianza de cada elemento

$$\alpha = \frac{n}{n-1} \left( 1 - \frac{\sum S_i^2}{S_x^2} \right) = \frac{2}{2-1} \left( 1 - \frac{10+11}{30} \right) = 0'6$$

↳ nº elementos del test

$$\beta = \frac{S_x^2 \cdot \sum S_i^2}{S_x^2 \left[ 1 - \sum \left( \frac{n_i}{n} \right)^2 \right]} = \frac{30 \cdot (10+11)}{30 \left[ 1 - \left( \frac{10}{25} \right)^2 + \left( \frac{15}{25} \right)^2 \right]} = 0'63$$

↳ nº total items  
items en cada subtest

Efectivamente,  
 $\alpha < \beta$

### EJERCICIO 9

$$r_{xe} = 0'4$$

$$S_x^2 = 25 \rightarrow S_x = \sqrt{25} = 5$$

$$X = 10$$

$$NC = 99\% \rightarrow \alpha = 0'01$$

$$Lim = X \pm E_{max} = 10 \pm 20 < \begin{matrix} 30 \\ -10 \end{matrix}$$

$$E_{max} = Se \cdot k = 2 \cdot 10 = 20$$

$$Se = S_x \sqrt{1 - r_{xx'}} = 5 \sqrt{1 - 0'84} = 5 \cdot 0'4 = 2$$

$$r_{xx'} = 1 - r_{xe}^2 = 1 - 0'4^2 = 1 - 0'16 = 0'84$$

$$k = \sqrt{\frac{1}{\alpha}} = \sqrt{\frac{1}{0'01}} = \sqrt{100} = 10$$

## EJERCICIO 10

$$r_{xx'} = 0.6$$

$$S_x = 5$$

$$NC = 95\% \rightarrow z_c = 1.96$$

$$X = 4$$

$$\text{Lim} = X \pm E_{\max} = 4 \pm 6.19 < \begin{matrix} 10.19 \\ -2.19 \end{matrix}$$

$$E_{\max} = z_c \cdot S_e = 1.96 \cdot 3.16 = 6.19$$

$$S_e = S_x \sqrt{1 - r_{xx'}} = 5 \sqrt{1 - 0.6} = 3.16$$

## EJERCICIO 11

$$N = 300$$

$$\bar{X} = 36$$

$$S_x^2 = 25 \rightarrow S_x = \sqrt{25} = 5$$

$$\frac{S_{xx'}}{S_x^2} = 0.81 = r_{xx'}$$

$$z_x = 1.5$$

$$NC = 99\% \rightarrow z_c = 2.58$$

$$(a) r_{xx'} = 0.81$$

$$r_{xv} = \sqrt{r_{xx'}} = \sqrt{0.81} = 0.9$$

$$(b) S_e = S_x \sqrt{1 - r_{xx'}} = 5 \sqrt{1 - 0.81} = 2.18$$

$$(c) \text{Lim} = V' \pm E_{\max} = 42.08 \pm 5.06 < \begin{matrix} 47.14 \\ 37.02 \end{matrix}$$

$$V' = r_{xx'} (X - \bar{X}) + \bar{X} = 0.81 (43.5 - 36) + 36 = 42.08$$

$$z_x = \frac{X - \bar{X}}{S_x} \rightarrow 1.5 = \frac{X - 36}{5} \rightarrow 1.5 \cdot 5 = X - 36$$

$$7.5 = X - 36$$

$$7.5 + 36 = X$$

$$43.5 = X$$

$$E_{\max} = z_c \cdot S_{vx} = 2.58 \cdot 1.96 = 5.06$$

Error típico de estimación  
d la población verdadera  $S_{vx} = S_e \sqrt{r_{xx'}} = 2.18 \sqrt{0.81} = 2.18 \cdot 0.9 = 1.96$

Obtenido en el apartado (b)

## EJERCICIO 12

TEST A	TEST B		Σ
	SOB.	NO SOB.	
SOB.	6 (a)	2 (b)	8 (g)
NO SOB.	0 (c)	4 (d)	4 (h)
Σ	6 (e)	6 (f)	12 (N)

1. HAMBLETON Y NOVICK:

$$p_0 = \frac{F_c}{N} = \frac{6+4}{12} = \frac{10}{12} = 0.83$$

2. KAPPA:

$$k = \frac{F_c - F_a}{N - F_a} = \frac{10 - 6}{12 - 6} = \frac{4}{6} = 0.67$$

$$F_a = \frac{6 \cdot 8}{12} + \frac{0 \cdot 4}{12} = \frac{48}{12} + \frac{0}{12} = 4 + 0 = 4$$

$$F_a = \frac{e \cdot g}{N} + \frac{f \cdot h}{N}$$